The Sampling Theorem

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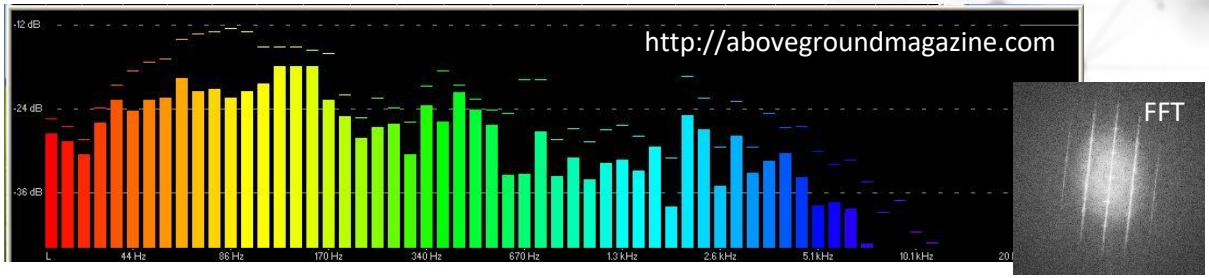
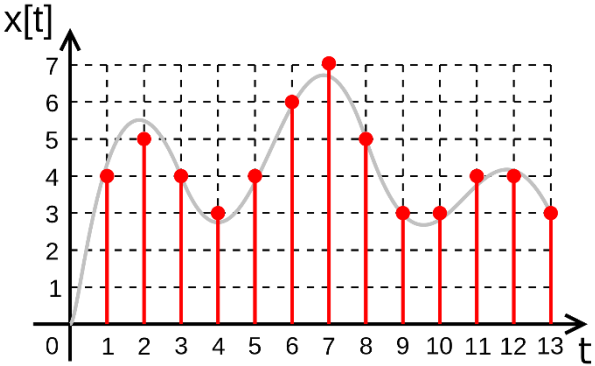
The central question for the digital acquisition of a measurement signal is the following: **Can I restore the original signal from the digitized measurement signal or not**? In other words: How large must the minimum **sampling frequency** be so that I can reconstruct the original signal from the digital values?

Analog-Digital-Converter



In electronics, an **analog-to-digital converter** (**ADC**, **A/D**, or **A-to-D**) is a system that converts an analog signal, such as a sound picked up by a microphone or light entering a digital camera, into a digital signal.

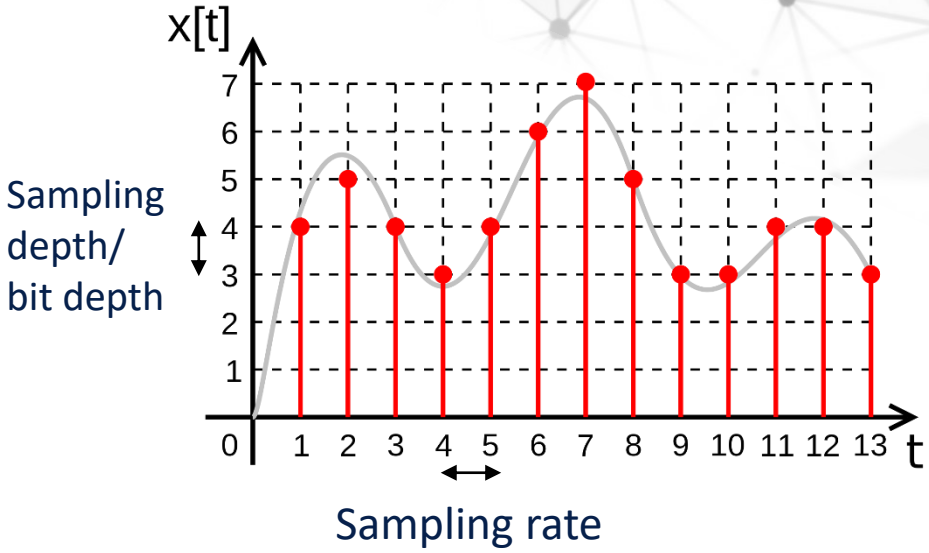


[wikipedia.com]

Sampling Depth and Rate

In signal processing, the **sampling depth/bit depth** is the accuracy with which the amplitude of an analog signal (also known as a time-continuous signal) can be reconstructed.

In signal processing, the **sampling rate** is the frequency with which an analog signal is sampled in a given time.

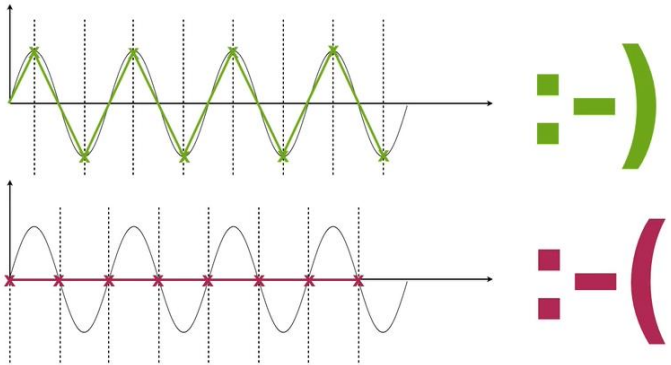


[wikipedia.com]

Nyquist-Shannon Sampling Theorem

The sampling theorem states that a signal band limited to f_{max} can **be reconstructed exactly** from a sequence of equidistant samples if it has been sampled at a frequency greater than $2 \cdot f_{max}$.

$$f_{sampling} > 2 f_{max}$$



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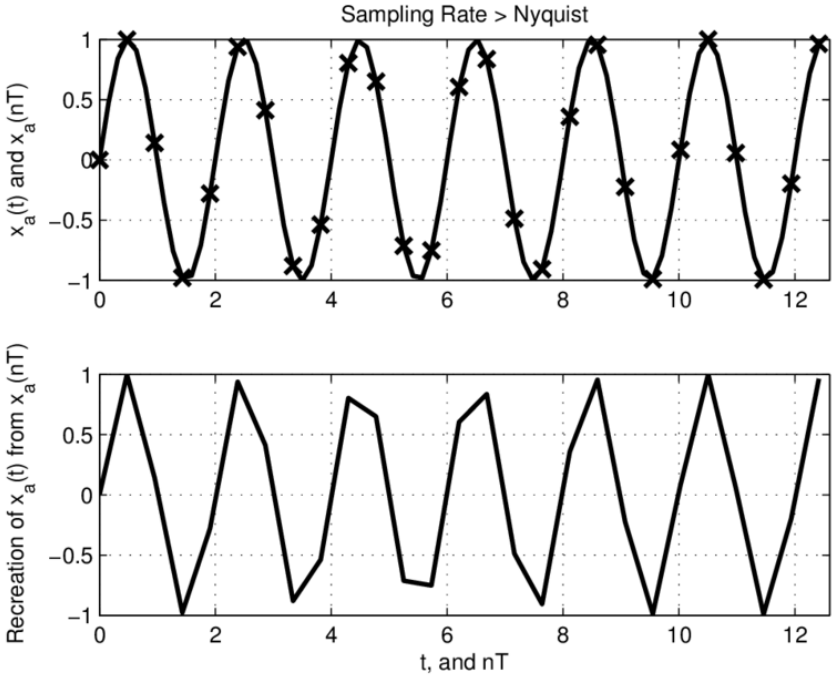


[youtube: sonic-vison.tv; Research gate: John Crassidis]

Nyquist Frequency

$$f_{nyquist} = \frac{1}{2} f_{sampling}$$

$$f_{signal} < f_{nyquist}$$

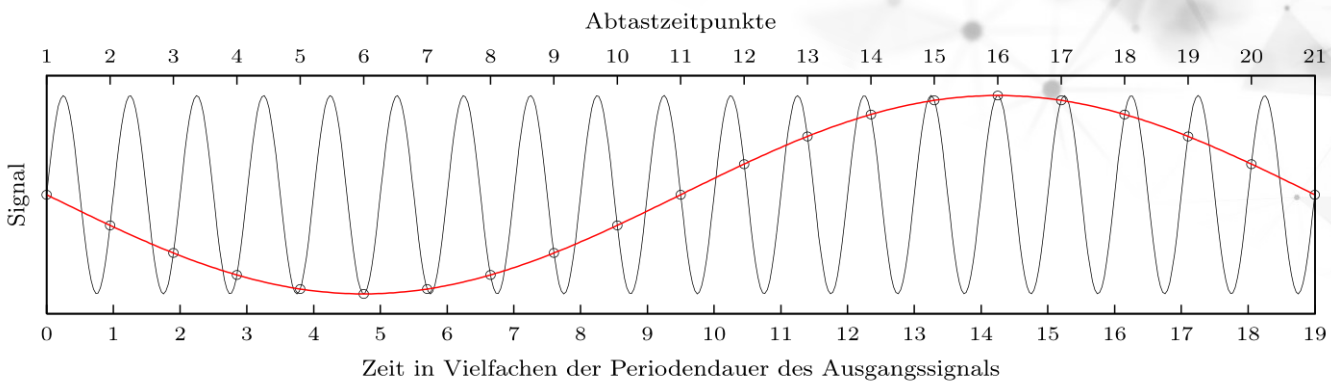
=

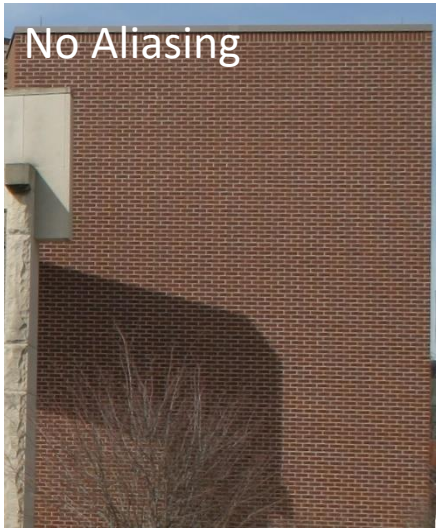
$$f_{sampling} > 2 f_{signal}$$

1889-1976

[wikipedia.com]

Under-Sampling / Aliasing

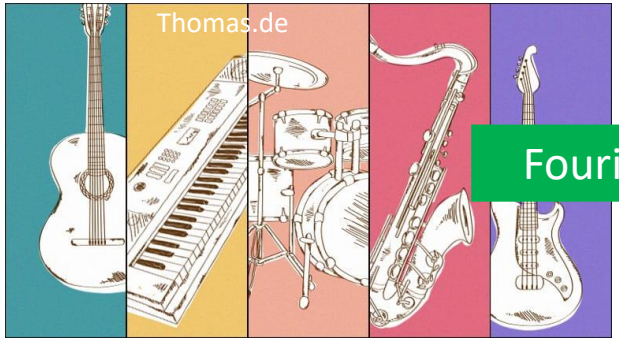




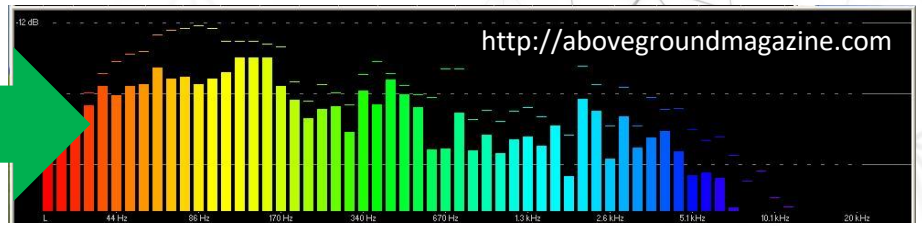
Alias effect occurs if frequencies are present that are larger than the **Nyquist frequency**.

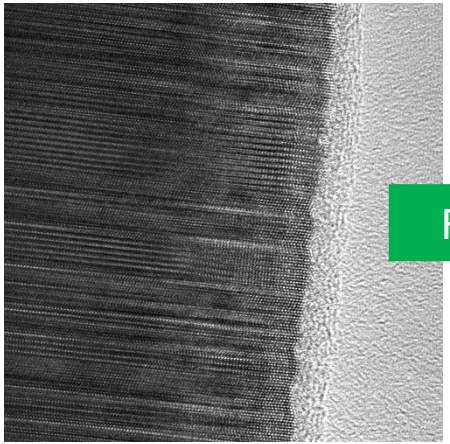
[wikipedia.com]

Fourier Transformation

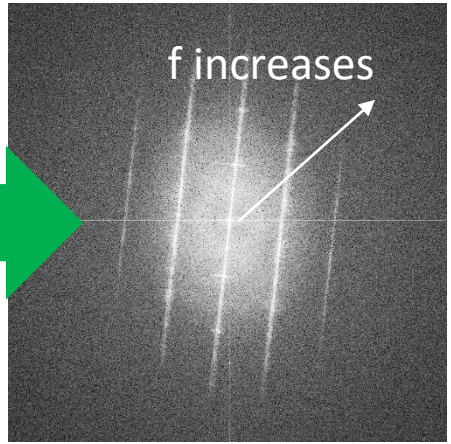


Fourier Trans.

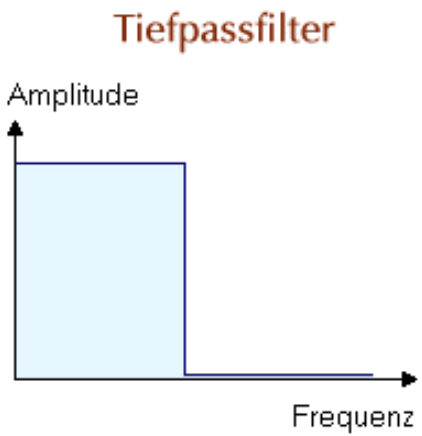


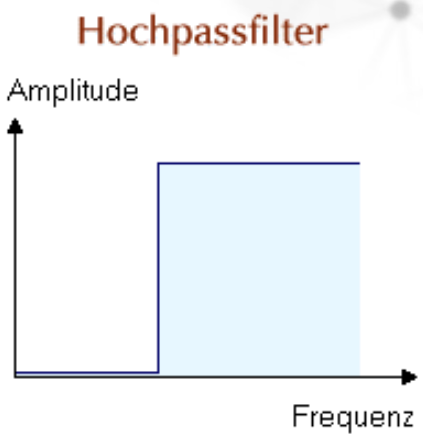


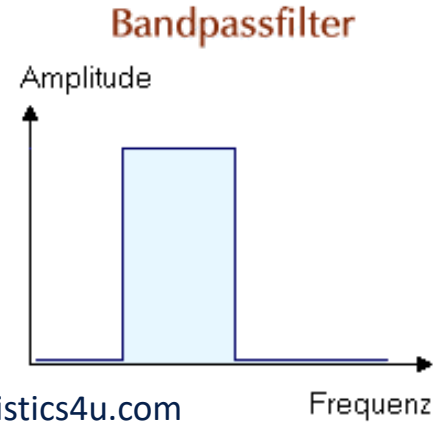
Fourier Trans.

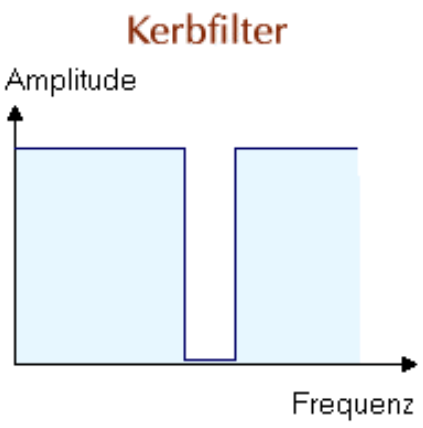


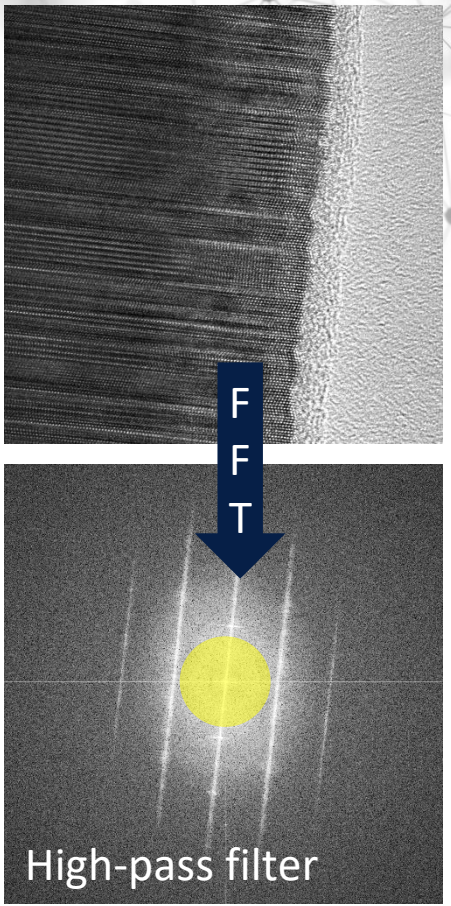
Filter





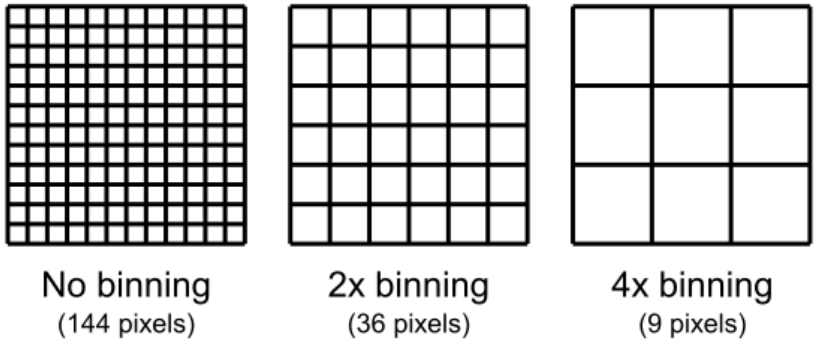




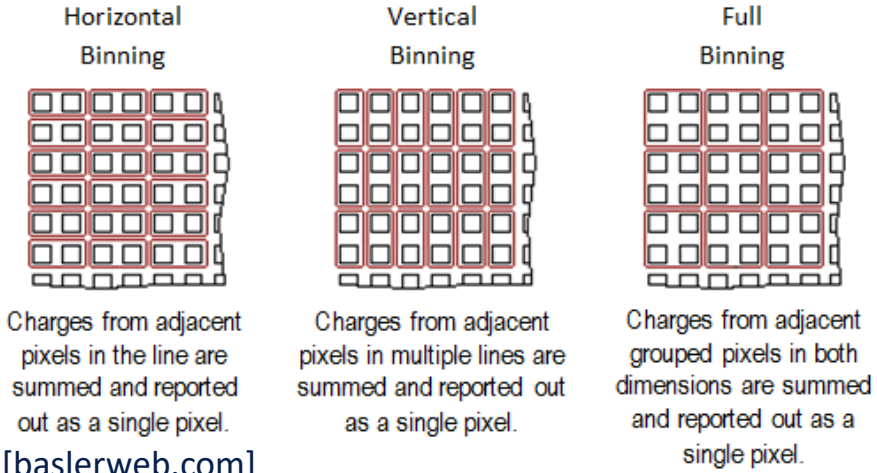


statistics4u.com

Binning



eyerys.com

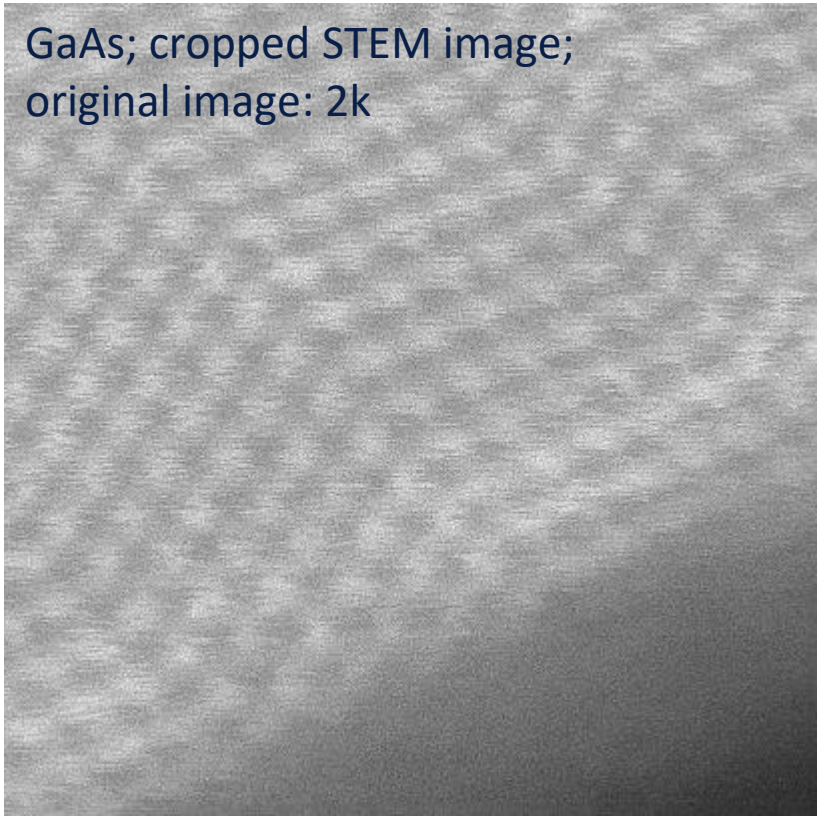


[baslerweb.com]

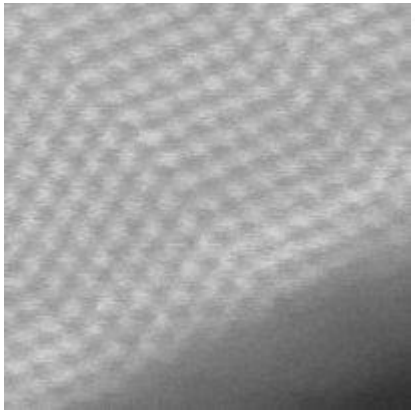
In the context of image processing, binning is the procedure of **combining a cluster of pixels** into a single pixel. As such, in 2x2 binning, an array of 4 pixels becomes a single larger pixel, reducing the overall number of pixels.

Binning: Size Effect

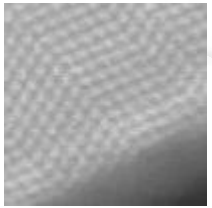
No binning, e.g. 1.0 s



2x2 binning, 0.25 s



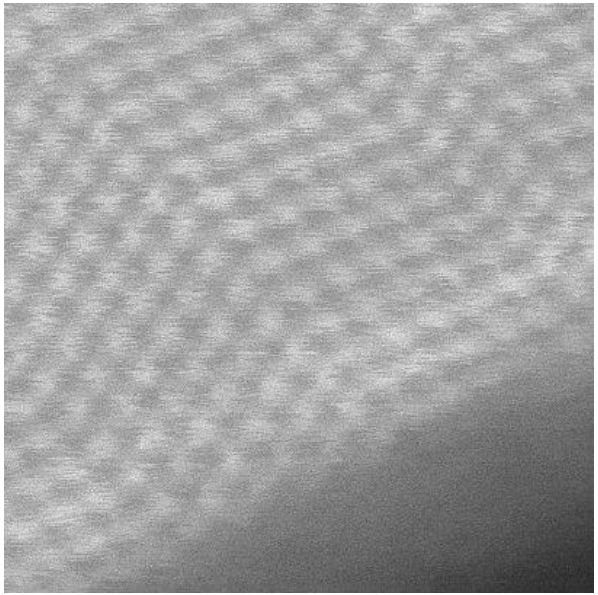
4x4 binning,
0.0625 s



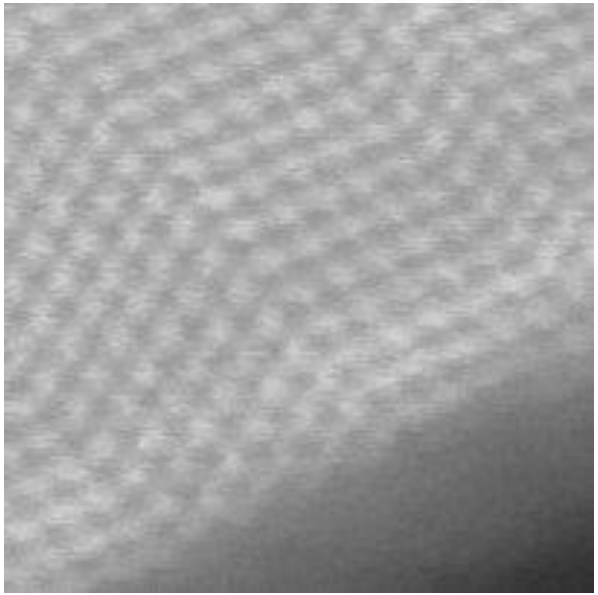
The **measurement time** (= dwell time) and **data size** also scale in a quadratic manner!

Binning: Effect on Noise and Resolution

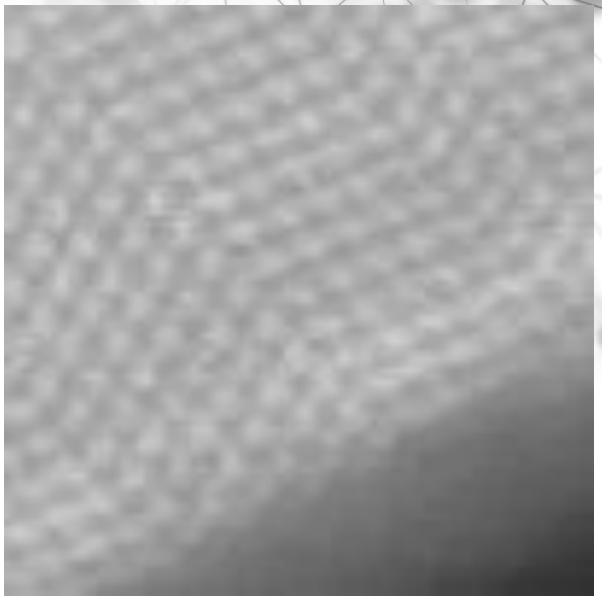
No binning



2x2 binning



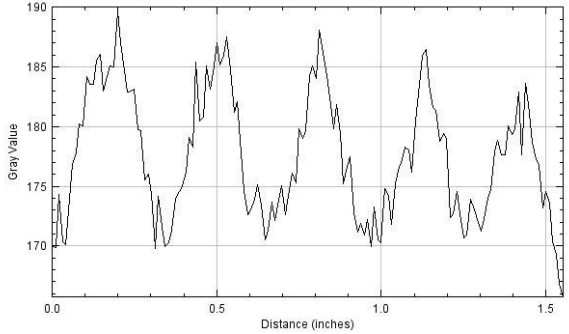
4x4 binning

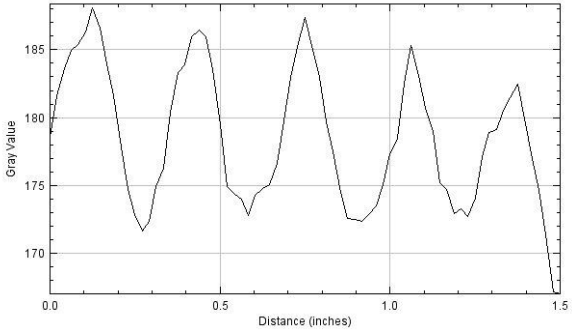


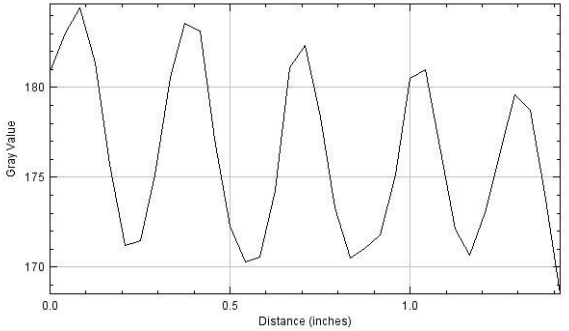
Noise

Resolution

Binning: Effect on Noise and Resolution







Noise

Resolution

EDX


Intensity (Counts)
Energy (keV)
Ti
C
O
Cu
TEASER → Next week

HAADF
Area #1
1 µm


SNICKERS